

Two approaches in the theory of atom interferometry

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In the theory of atom interferometry (AI) [1], one can choose between several approaches. I use the approach based on the density matrix equation in the Wigner representation (DMEWR) [2]. Starting from the article [3], this is a traditional approach in the theory of phenomena related to the quantization of the atomic center mass motion and laser cooling [4]. However, to my knowledge, it has been used sparingly in the context of AI [5, 6]. The convenience of this approach is that, for the time between Raman pulses, the density matrix obeys an equation that is similar to the classical Liouville equation for the distribution function [2].

Another approach here is path integrals (PI) [7], which is used starting from article [8].

I am here going to compare these 2 approaches to a specific problem, the part of the AI phase caused by an external test mass. The expression for this part is used in measurements of the Newtonian gravitational constant G using AI. This is a promising technique. Using a recently elaborated [9] error model, I showed that at the current state of the art in the AI [10–13], this measurement should be off 200ppb accuracy, which is more than 2 orders of magnitude better than the accuracy accepted now [14]. I justified the expression for the phase by applying DMEWR approach [15]. While in real experiments [16, 17] the PI approach was used. The PI approach has also been used in article [18].

I am going to analytically compare these two approaches here. Consider atoms moving in the gravity potential

$$V(\vec{x}) = -M_a \vec{g} \cdot \vec{x} + \delta V(\vec{x}), \quad (1)$$

where M_a is the atomic mass, \vec{g} is Earth's gravity field, $\delta V(\vec{x})$ is the gravity potential of the external test mass, which we assume to be small,

$$\delta V(\vec{x}) \ll V(\vec{x}). \quad (2)$$

After interacting with $\pi/2 - \pi - \pi/2$ sequence of Raman fields, the atomic levels' populations acquire interferometric terms whose phase contains the addition $\delta\phi$, caused by the external test mass gravity field. Using DMEWR approach, we calculated [15] $\delta\phi$ for the arbitrarily moving test mass. In the case of the stationary test mass, which we consider here, from Eqs. (58a,62,64,73,88) in article [15], one finds

$$\delta\phi = \vec{k} \cdot \int_0^T dt \left\{ t \delta \vec{g} \left[\vec{s}(t_1 + t) + \frac{\hbar \vec{k}}{2M_a} t \right] + (T - t) \delta \vec{g} \left[\vec{s}(t_1 + T + t) + \frac{\hbar \vec{k}}{2M_a} (T + t) \right] \right\} + \phi_Q, \quad (3a)$$

$$\phi_Q \approx \frac{\hbar^2}{24M_a^2} k_i k_j k_l \int_0^T dt \left\{ t^3 \chi_{ijl} [\vec{s}(t_1 + t)] + (T - t)^3 \chi_{ijl} [\vec{s}(t_1 + T + t)] \right\}, \quad (3b)$$

$$\vec{s}(t) = \vec{v}t + \frac{1}{2} \vec{g}t^2, \quad (3c)$$

where t_1 is the time delay between the time of the atoms' launching ($t = 0$) and the 1st Raman pulse, T is the time separation between Raman pulses,

$$\delta \vec{g}(\vec{x}) = -\frac{1}{M_a} \partial_{\vec{x}} \delta V(\vec{x}), \quad (4a)$$

$$\chi_{ijl}(\vec{x}) = \partial_{x_j} \partial_{x_i} \delta g_i(\vec{x}) \quad (4b)$$

are the acceleration and the curvature tensor of the test mass field. In Eqs. (3) we assumed, for simplicity, that the atoms are launched from the frame origin. One can expand $\delta\phi$ into a power series over the Planck constant. Since only the 1st nonzero part of Q-term (3b) has been obtained in [15], one should truncate the series for $\delta\phi$ with a second order term. Using that

$$\delta g_i(\vec{x} + \vec{\varepsilon}) \approx \delta g_i(\vec{x}) + \gamma_{ij}(\vec{x}) \varepsilon_j + \frac{1}{2} \chi_{ijl}(\vec{x}) \varepsilon_j \varepsilon_l, \quad (5)$$

where

$$\gamma(\vec{x}) = \partial_{\vec{x}} \delta \vec{g}^T(\vec{x}) \quad (6)$$

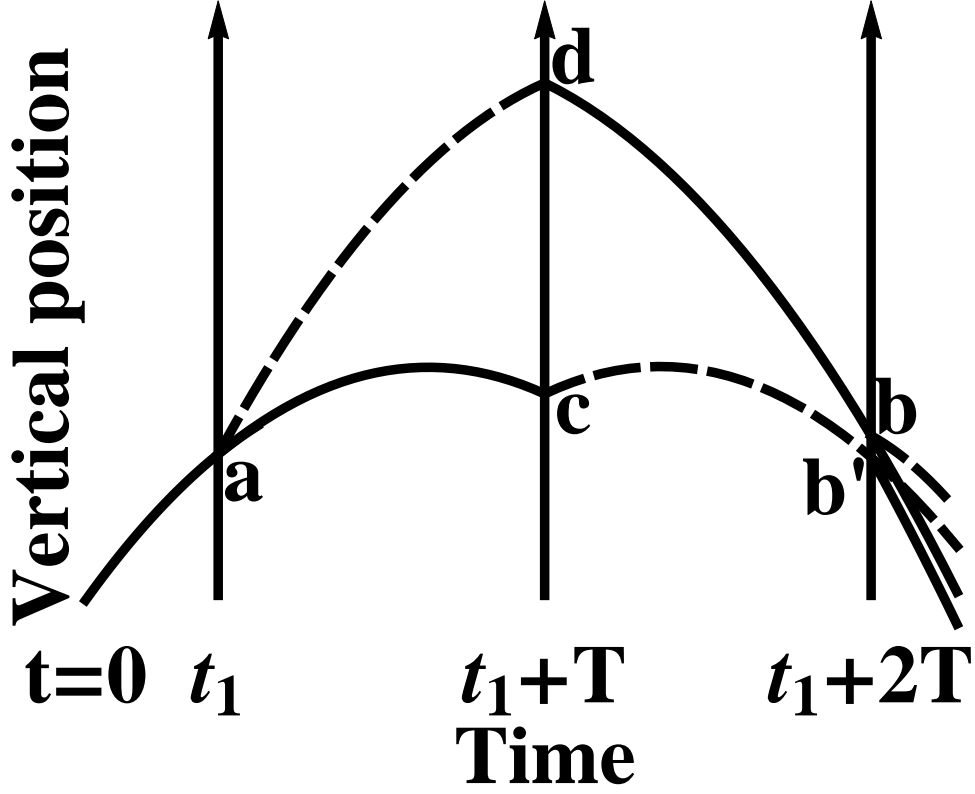


FIG. 1: Atomic trajectories in the gravity field (1) after interacting with three Raman pulses at moments $\{t_1, t_1 + T, t_1 + 2T\}$. Solid and dashed curves correspond to the ground and excited atomic states.

is the gravity-gradient tensor, and the equality

$$T^3 - t^3 = \frac{1}{4} \left[(T - t)^3 + 3 (T - t) (T + t)^2 \right], \quad (7)$$

one obtains the following series

$$\begin{aligned} \delta\phi = & \vec{k} \cdot \int_0^T dt \{ t \delta \vec{g} [\vec{s}(t_1 + t)] + (T - t) \delta \vec{g} [\vec{s}(t_1 + T + t)] \} \\ & + \frac{\hbar}{2M_a} \vec{k} \cdot \int_0^T dt \{ t^2 \underline{\chi} [\vec{s}(t_1 + t)] + (T^2 - t^2) \underline{\chi} [\vec{s}(t_1 + T + t)] \} \vec{k} \\ & + \frac{\hbar^2}{6M_a^2} k_i k_j k_l \int_0^T dt \{ t^3 \chi_{ijl} [\vec{s}(t_1 + t)] + (T^3 - t^3) \chi_{ijl} [\vec{s}(t_1 + T + t)] \}. \end{aligned} \quad (8)$$

Consider now the PI approach. In this approach, one can calculate the phase in 2 different ways [19, 20], using the expression

$$\phi = \phi_{prop} + \phi_{laser} + \phi_{sep}, \quad (9)$$

where ϕ_{prop} , ϕ_{laser} , ϕ_{sep} are the so-called propagation, laser and separation [20] phases, or using an approximate expression for the addition to phase [19], valid at condition (2)

$$\delta\phi = -\frac{1}{\hbar} \oint_{adbca} dt \delta V [\vec{x}_0(t)], \quad (10)$$

where $\vec{x}_0(t)$ is the atom trajectory in the absence of test-mass potential (see Fig. 1).

Expression (9) was used in experiment [16], while Eq. (10) was used in experiment [17] and article [18]. The equivalence of the Eqs. (9) and (10) has been verified for the linear gravity potential [19]. After making some calculations, I also verified this equivalence for arbitrary small nonlinear stationary test mass potential [21].

For trajectories shown in Fig. 1, one gets

$$\begin{Bmatrix} \vec{x}_0(t)_{ad} \\ \vec{x}_0(t)_{ac} \\ \vec{x}_0(t)_{db} \\ \vec{x}_0(t)_{cb'} \end{Bmatrix} = \vec{s}(t) + \frac{\hbar \vec{k}}{M_a} \begin{pmatrix} t - t_1 \\ 0 \\ T \\ t - t_1 - T \end{pmatrix}, \quad (11)$$

where $\vec{s}(t)$ is given by Eq. (3c). Using these trajectories, one can represent the contour integral in Eq. (10) as 2 integrals from t_1 to $t_1 + T$ and from $t_1 + T$ to $t_1 + 2T$. Transforming both integrals to those from zero to T , one finds

$$\delta\phi = -\frac{1}{\hbar} \int_0^T dt \left\{ \delta V \left[\vec{s}(t_1 + t) + \frac{\hbar \vec{k}}{M_a} t \right] - \delta V [\vec{s}(t_1 + t)] + \delta V \left[\vec{s}(t_1 + T + t) + \frac{\hbar \vec{k}}{M_a} T \right] - \delta V \left[\vec{s}(t_1 + T + t) + \frac{\hbar \vec{k}}{M_a} t \right] \right\}. \quad (12)$$

Finally, using for potential expansion

$$\delta V(\vec{x} + \vec{\varepsilon}) \approx \delta V(\vec{x}) - M_a \left[\delta \vec{g}(\vec{x}) \vec{\varepsilon} + \frac{1}{2} \vec{\varepsilon} \cdot \underline{\gamma}(\vec{x}) \vec{\varepsilon} + \frac{1}{6} \chi_{ijl}(\vec{x}) \varepsilon_i \varepsilon_j \varepsilon_l \right] \quad (13)$$

one arrives at the series (8).

The coincidence of the power series for the phase $\delta\phi$ in DMEWR- PI-approaches means that Q-term [15] has to be included in DMEWR-approach, while no Q-term arises in PI-approach.

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